A fully abstract semantics for concurrent graph reduction

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AB AC – s p p r pr s nts u str t s nt s or v r nt t unt p λ u us t r urs v r t ons – rst pr s nt su r st n or on u str t on or t un t p λ u us on ntr t n on AB A Y n G s or on t λ u us– AB A Y n G s or s s on t ost out r ost r u t on t out s r n – s s not n nt n n p nt t ons o s r n r u n s nt

1 Introduction

s p p r s out t r t ons p t n t o s o put r s n *full abstraction* n *concurrent graph reduction* Fu str t on s t stu o r t n not t on n op r t on s nt s Con urr nt r p r u t on s n nt p r p nt t on t n qu or non str t un t on pro r n n u s

nt s p p r pp t t n qu s o AB A Y n G to pr s nt u str t not t on s nt s or t on urr nt r p r u t on or t v n n EY E s t t oo

n o n so us t o s ro u str t on o p r p nt t on n on urr n t or

1.1 Full abstraction

Fu str t on or n n E p or s t r t ons p

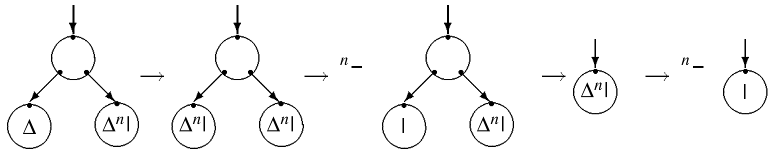
t s v op AD H s n p nt t on t ost out r ost r u t on—H o s r t t t ost out r ost r u t on nt pon nt t to u t n pr ss on, u to oss *sharing* nor t on—For p, n

$$\mathsf{I} = \lambda x.x \qquad \Delta = \lambda x.xx \qquad M N = N \qquad M^{n+} N = M(M^n N)$$

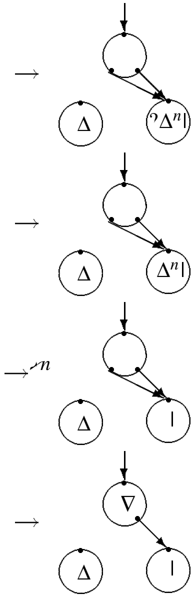
n t v u t on $\Delta^{n+}\,\mathsf{I} \to^* \mathsf{I}$ s

$$\Delta^{n+}\,\mathsf{I} \to (\Delta^n\,\mathsf{I})(\Delta^n\,\mathsf{I}) \to^{n-} \mathsf{I}(\Delta^n\,\mathsf{I}) \to \Delta^n\,\mathsf{I} \to^{n-} \mathsf{I}$$

us, $\Delta^n \mathsf{I}$ t s $^{n}-$ r u t ons to t r n t — s pon nt o up s us op n $\Delta^n \mathsf{I}$ n t r u t on $\Delta^{n+}\,\mathsf{I} \to (\Delta^n\,\mathsf{I})(\Delta^n\,\mathsf{I})$, n n r s n r t s nt tr s ort s r u t on, r not s un t on pp t on



s n n s us t p nt t on β r u t on t su st tu t on— n r u $(\lambda w.M)N \to M[N/w]$, s p r t op N or o urr n w n M, n op t n s to r u s p r t — n r o t s n n r t r t n op n t r s, op *pointers to* t r s, t t s r u s nt *graphs* r t r t n s nt *trees*—For p, t



up t n

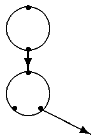
p n tr v rs

n u t on

up t n

$\longrightarrow$

not *con uent* or *Church Rosser* , s n sp n tr v rs



- G r o t on s s nt un port nt, so r p n on r t n on r t out r o t n. n ou p tt s to tru s n r o t on s ntro u on us or t t ons.
- n s s nt un port nt, so r p n on r rr sp t t r ts no s r t or not. n p rt u r, t s ns t t on urr nt u t on s s nt qu nt to s qu nt u t on.
- r nt tr nsp r n , ns t t t s s nt un port nt r p ont ns op no , or po nt r to no .

r r nu r pp t ons or u str t s nt s

FY G C ZA . A nu r o p rs non str t un t on n u s, not H s o p r or t *G* n , us opt t ons. n opt rs, not *peephole optimizers* EY E , C . r p on s t r t not r s nt qu nt, ut or nt t r . s nt s s orr t, t n no t t n su opt t on t s op r t on our n ont ts.

n ortun t , t s nt s s not o p t, t n t r op t t ons t t r not s nt qu nt, n t r s t pt t on or t o p r r t r to us *ad hoc* r son n to ust s nt n opt t on, on t roun s t t s nt s s too n . t s nt s s u

2 Tree reduction

s C pt r pr s nts su r st n or on u str t o s or t ost out r ost r u t on t unt p λ u us− t on ntr t s on AB A Y n G s or on t λ u us, ut so n u s t r ro AB A Y , BA E D EG , BA A D EG *et al.* B D , E CE n −

2.1 The λ-calculus with P

n t s C pt r, s uss t t or v op AB A Y n G, s on *leftmost outermost* r u t on− s s t s nt s s o t *non strict* un t on n u s su s A G s , FA B s on r, E s Go r, E s r n , n H s H DA *et al.*, −

n t unt p λ u us, pr ss ons r un t ons, n t s un t ons t un t ons s nputs, n r turn ot r un t ons− n r r t s s pur t or o put t on, str t ro ons r t ons o t − unt p λ u us s t r or s o pr ss on

- A *free variable* x−
- An *application* MN−
- An *abstraction* $\lambda x\,.\,M$−

u t r s r *sequential* n t on or o put t on s β r u t on, r n str t on s pp $(\lambda x\,.\,M)N \to M[N/x]$−Fo o n ou p t t t n n u str t s nt s u s p r so or p r o put t on− r r nu r poss p r o n tors on n us p r on t on , AB A Y n G us p r on r n , n B D s n

ours v s to ω *continuous* un t ons t t s

$$a \text{ s t } \quad \text{ t } a \leq a \leq \cdots$$

t n

$$fa \text{ s t } \quad \text{ t } fa \leq fa \leq \cdots$$

For p t s rs t odd un t on s n

$$- \text{ s t } \quad \text{ t } \leq - \leq \quad \leq \cdots$$

ut

$$\text{ s not t } \quad \text{ t } - \leq \quad \leq - \leq \cdots$$

n t not t on s nt s Λ_P n $\mathbf{D} \simeq (\mathbf{D} \to \mathbf{D})_\perp$ o s o t t su **D** ust st pr s nt t s t t s qu n n t o ns $\mathbf{D}, \mathbf{D}, \ldots$ r

$$\mathbf{D} = \qquad \mathbf{D}_{n+} = (\mathbf{D}_n \to \mathbf{D}_n)_\perp$$

s n so pr s nt s t po nt *functor* F t n o ns

$$F\mathbf{D}_i = (\mathbf{D}_i \to \mathbf{D}_i)_\perp = \mathbf{D}_{i+}$$

n n or r to s o t t **D** sts s o t t F s ont nuous n or r to o t s pr s nt

- A not on *domain* su t t t on po nt o n s o n n F s un tor t n o ns
- A not on *order* t n o ns t st nt n r v r n o ns s t
- A not on *continuous functor* t n o ns su t t F s ont nuous

Fo o n us t *category of* ω *cpo s with embeddings* s t ppropr t not on or r o ns n F s ont nuous un tor t ust v st po nt us s our n t on **D**
r st t s s t on pr s nt t t n t s t s onstru t on s n t s ort r n r so s p t or t or nt r st

EXA E ¬lift C → $C_\perp$ s un tor s n v

- n o t lift A in $C_\perp$ or A A

rro e^R s un qu n, so e $A \to B$ in CPOE n f $B \to A$ in ωCPOE t n

$$(e \circ f \leq \mathrm{id}, f \circ e = \mathrm{id}) \quad \text{p s} \quad e^R = f$$

$()_\perp$ ωCPOE → ωCPOE s t t n un tor t

- $A_\perp$ in ωCPOE or A in ωCPOE
- $e_\perp$ $A_\perp \to B_\perp$ in ωCPOE or e $A \to B$ in ωCPOE

Δ ωCPOE → ωCPOE s t on un tor t

- $\Delta A = (A, A)$ in ωCPOE or A in ωCPOE
- $\Delta f = (f, f)$ $\Delta A \to \Delta B$ in ωCPOE or f $A \to B$ in ωCPOE

$(\to)$ ωCPOE → ωCPOE s t ω ont nuous un t on sp un tor t

- $(A \to B)$ in ωCPOE or (A, B) in ωCPOE
- $(e \to f)$ $(A \to B) \to (A' \to B')$ in ωCPOE or (e, f) $(A, B) \to (A', B')$ in ωCPOE

r $e \to f$ s n

$$(e \to f)g = f \circ g \circ e^R$$

$$(e \to f)^R g = e \circ g \circ f^R$$

s t n t o t n ωCPOE □

DEF A o on $\{e_i \ A_i \to A \text{ in ωCPOE} \mid i \text{ in } \omega\}$ s *determined* $\bigvee\{e_i \circ e_i^R \mid i \text{ in } \omega\} = \mathrm{id}$ □

Any determined cocone is a colimit

F t $\{e_i \ A_i \to A \mid i \text{ in } \omega\}$ t r n o on n ω n $\{e_i^j \ A_i \to A_j \mid i \leq j \text{ in } \omega\}$ n or n ot r o on $\{f_i \ A_i \to B \mid i \text{ in } \omega\}$, n g $A \to B$ s

$$g = \bigvee\{f_i \circ e_i^R \mid i \text{ in } \omega\}$$

$$g^R = \bigvee\{e_i \circ f_i^R \mid i \text{ in } \omega\}$$

n n s o t t g s t un qu n su t t $g \circ e_i = f_i$ us $\{e_i \ A_i \to A \mid i \text{ in } \omega\}$ s o t □

Any ω chain in ωCPOE has a determined cocone

F t $\{e_i^j \ A_i \to A_j \mid i \leq j\}$ n ω n An *instantiation* t s n s un t on f su t t

$$\mathrm{dom}\, f = \omega \qquad fi \in A_i \qquad e_i^{jR}(fj) = fi$$

t n n

$$A = \{f \mid f \text{ s n nst nt t on}\}$$

t t po nt s or r n s s n ω po t on

$$\bigvee\{f_i \mid i \text{ in } \omega\} j = \bigvee\{f_i j \mid i \text{ in } \omega\}$$

n n

$$e_i a j = \begin{cases} e_i^j a & i \leq j \\ e_j^{iR} a & \text{ot r s} \end{cases}$$

$$e_i^R f = fi$$

n s o t t $\{e_i \ A_i \to A \mid i \text{ in } \omega\}$ s t r n o on □

DEF **D** s t t r n o t t ω n

$$\mathbf{D} =$$

$$\mathbf{D}_{i+} = (\mathbf{D}_i \to \mathbf{D}_i)_\perp$$

t e_i $\mathbf{D}_i \to \mathbf{D}$ in ωCPOE n ropos t on n **D** s t n t po nt t un tor $()_\perp \circ (\to) \circ \Delta$ n ropos t on □

2.6 Logical presentation of D

n t on n str t pr s nt t on **D**, us n t t or ω po s t n nt s s t on, pro on r t pr s nt t on **D**, s r to C s *information systems* Fo o n AB A Y s *domain theory in logical form* us t pro o Φ s n t rn t pr s nt t on **D** n p rt u r, s o t t t ω po *lters* Φ s qu nt to **D**

DEF Ψ ⊆ Φ s *lter*

- ω ∈ Ψ
- φ ∈ Ψ n ⊢ φ ≤ ψ t n ψ ∈ Ψ
- φ, ψ

- $\vdash \phi \leq \psi$ [illegible] $[\![\phi]\!] \leq [\![\psi]\!]^{-}$
- a s ω

−Fo o s ro t n t on pt −

− $a = \perp$ t n

$$a = \perp = \bigvee \emptyset = \bigvee\{b \mapsto c \mid b \mapsto c \le a\}$$

t r s , n s o t t or n d

$$\mathsf{apply}\, a d = \mathsf{apply}(\bigvee\{b\mapsto c \mid b\mapsto c\le a\})d$$

n so $a = \bigvee\{b\mapsto c \mid b \mapsto c \le a\}$ −

− $a \mapsto b \le \bigvee C$ or n ω n $C \subseteq \mathbf{D}$, t n

$$b = \mathsf{apply}(a\mapsto b)a \le \mathsf{apply}(\bigvee C)a = \bigvee\{\mathsf{apply}\, ca \mid c\in C\}$$

n b s ω o p tt r s $c \in C$ su t t $b \le \mathsf{apply}\, ca$ so

$$a\mapsto b \le a \mapsto \mathsf{apply}\, ca \le c$$

us $a\mapsto b$ s ω o p t −

− $a\mapsto b \le \bigvee A$ or n t s t $A \subseteq \mathbf{D}$, t n

$$b = \mathsf{apply}(a\mapsto b)a \le \mathsf{apply}(\bigvee A)a = \bigvee\{\mathsf{apply}\, ca \mid c\in A\}$$

n b s pr , t r s $c \in A$ su t t b

$\Rightarrow \nu x\,.\,\Gamma \vdash \lambda x\,.\,M \quad \psi_i \rightarrow \chi_i$ $\rightarrow$I

$\Rightarrow \Gamma \vdash \lambda x\,.\,M \quad \psi_i \rightarrow \chi_i$ $\leq$

us (∧I) n (≤), $\Gamma \vdash \lambda x\,.\,M \quad \phi^-$ □

s st to t r t not t on n proo t or t pr s nt t ons o t o , n n st rt to n t s t t op r t on pr s nt t on− o n t , s o t t t not t on s nt s r sp ts t op r t on s nt s o o n BA E D EG s n t on o λ *theory*

$(M \sqsubseteq_D N \Rightarrow M \sqsubseteq_S N)$ For n Γ n ϕ, $M \sqsubseteq_D N$ t n

$$
\begin{array}{ll}
\Gamma \vdash M \quad \phi & \\
\quad \Rightarrow [\![\phi]\!] \leq [\![M]\!][\![\Gamma]\!] & \text{ropn} \\
\quad \Rightarrow [\![\phi]\!] \leq [\![N]\!][\![\Gamma]\!] & \text{H pot s s} \\
\quad \Rightarrow \Gamma \vdash M \quad \phi & \text{ropn}
\end{array}
$$

us $M \sqsubseteq_D N$ t n $M \sqsubseteq_S N$ —

$(M \sqsubseteq_S N \Rightarrow M \sqsubseteq_D N)$ For n σ, $M \sqsubseteq_S N$ t n

$$
\begin{array}{l}
[\![M]\!]\sigma \\
\quad = \bigvee \{ [\![\phi]\!] \mid [\![
\end{array}
$$

- rec D in M s *recursive declaration* ❧ D n M^-

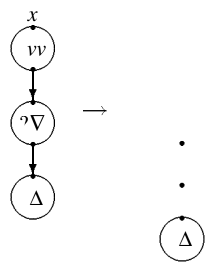
EXA E –

- $x = M$,

pp t on of M to ts , t s r n n r n

$$
\begin{aligned}
x &= u \quad v,\\
u &= \nabla z,\\
v &= ?\nabla z,\\
z &= ?M
\end{aligned}
$$

ζ p



DEF ◥ $\neg\mapsto$ s v n o s

$$
\begin{array}{ll}
(\textsc{build}) & x \;=\; (\mathsf{rec}\,D\,\mathsf{in}\,M) \mapsto \mathsf{local}\,D\,\mathsf{in}\,(x \;=\; M) \\
(\nabla\textsc{trav}) & x \;=\; \nabla y, y \;=\; ?M \mapsto x \;=\; \nabla y, y \;=\; M \\
(\quad\textsc{trav}) & x \;=\; y \quad z, y \;=\; ?M \mapsto x \;=\; y \quad z, y \;=\; M \\
(\vee\textsc{trav}) & x \;=\; y \vee z, y \;=\; ?M \mapsto x \;=\; y \vee z, y \;=\; M \\
(\nabla\textsc{upd}) & x \;=\; \nabla y, y \;=\; \lambda w.\, M \mapsto x \;=\; \lambda w.\, M, y \;=\; \lambda w.\, M \\
(\quad\textsc{upd}) & x \;=\; y \quad z, y \;=\; \lambda w.\, M \mapsto x \;=\; M[z/w], y \;=\; \lambda w.\, M \\
(\vee\textsc{upd}) & x \;=\; y \vee z, y \;=\; \lambda w.\, M \mapsto x \;=\; \mathsf{I}, y \;=\; \lambda w.\, M \\
(\gamma) & \nu(\mathsf{wv}\,D)\,.\,D \mapsto \varepsilon
\end{array}
$$

n stru tur ru s

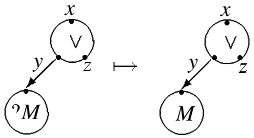
$$
(\textsc{l})\ \frac{D \mapsto E}{D, F \mapsto E, F} \qquad (\textsc{r})\ \frac{D \mapsto E}{F, D \mapsto F, E} \qquad (\nu)\ \frac{D \mapsto E}{\nu x\,.\,D \mapsto \nu x\,.\,E}
$$

ot t t $D \mapsto E$ t n $\mathsf{rv}\,D \supseteq \mathsf{rv}\,E$ n $\mathsf{wv}\,D = \mathsf{wv}\,E$−

- $D \to E$ $D \equiv\mapsto\equiv E$−
- $D \to E$ $D \equiv E$, n $D \to^{n+} E$ $D \to\to^{n} E$−
- $D \to^{*} E$ $\exists n\,.\,D \to^{n} E$−
- $D \to^{\leq i} E$ $\exists n \leq i\,.\,D \to^{n} E$− □

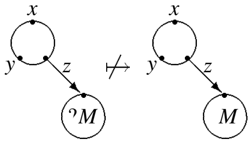
EXA E

$$x = y\vee z, y = {}^{?}M \mapsto x = y\vee z, y = M$$

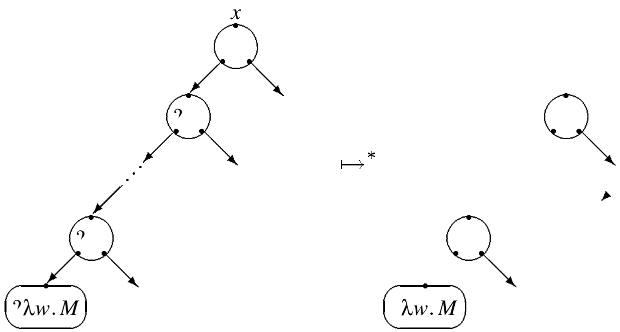


ot t t s n r o n v u t on, v

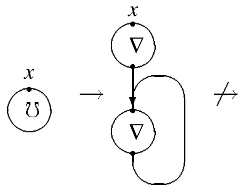
$$x = y \quad z, z = {}^{?}M \not\mapsto x = y \quad z, z = M$$



s p s s sp n tr v rs us t r t n sp n un t n r t on, pp t on, n or no s, t For p



Ho v r, t o or r o t on nvo v s r p so r tr r s ,
n so s u r r r nu r t , n so ss s op or on urr n – n p


x
x
∇
∇

$$\mathsf{set}\, X\, f\, g\, \sigma\, x = \begin{cases} f(g\sigma)x & x \in X \\ \sigma x & \text{ot r s} \end{cases}$$

$$\mathsf{fix}\, f = \bigvee \{ f^n \bot \mid n \;\mathsf{in}\; \omega \}$$

- $M \sqsubseteq_D N$ $[\![M]\!] \leq [\![N]\!]$ −
- $D \sqsubseteq_D E$ $\mathsf{wv}\, D = \mathsf{wv}\, E$ n $[\![D]\!] \leq [\![E]\!]$ − □

EXA E − ns o t tt s nt s t o r t r s ⊤ s n

$[\![\mathsf{rec}\, x = \lambda y.\, \nabla x \;\mathsf{in}\; \nabla x]\!]$

$\quad = [\![\nabla x$

y s t s s ψ t n

$$w = \lambda y.w, x = \lambda y.w, z = x \quad y, D$$
$$\rightarrow w = \lambda y.w, x = \lambda y.w, z = \nabla w, D$$
$$\rightarrow w = \lambda y.w, x = \lambda y.w, z = \lambda y.w, D$$

n u t ons s t s s z χ. us

$$(w = \lambda y.w, x = \lambda y.w) \quad \phi$$

Fro t s t s s p to s o t t $(w = \lambda y.w)$ $(w \quad \phi)$.

s n t on p n s on t not on r p t ns on, s t pr or r $D \sqsubseteq E$.

DEF .D $\sqsubseteq$ E n n $\vec{x}, \vec{y}, D'$ n E' su t t

$$D \equiv \nu\vec{x}.D' \qquad E \equiv \nu\vec{x}\vec{y}.(D', E') \qquad \mathrm{fv}\, D \cap \vec{y} = \emptyset$$

ot t t $\sqsubseteq$ s pr or r, n t t $D \sqsubseteq E \sqsubseteq D$ $D \equiv E$. □

n t n n t t op r t on nt rpr t t on t o .

DEF .For os r t ons, $\models D \quad \Delta$ s n t o s

$$(\varepsilon\mathrm{I}) \quad \models D \quad \varepsilon \qquad (\omega\mathrm{I}) \quad \models D \quad (x \quad \omega)$$

n stru tur ru s

$$(\wedge\mathrm{I})\ \frac{\models D \quad \Gamma \qquad \models D \quad \Delta}{\models D \quad \Gamma \wedge \Delta} \qquad (\rightarrow\mathrm{I})\ \frac{\forall (z = x \quad y) \sqsubseteq E \sqsupseteq D. \quad D\Downarrow_x \qquad \models E \quad (y \quad \phi) \Rightarrow \models E \quad (z \quad \psi)}{\models D \quad (x \quad \phi \rightarrow \psi)}$$

s n n r to n D n n $\Gamma \models D \quad \Delta$

$$\forall E.(\models D, E \quad \nu(\mathrm{wv}\, D).\Gamma) \quad \text{p} \quad \text{s}\,(\models D, E \quad \Delta)$$

r , $\Gamma \models M \quad \phi$

$$\forall D, z.(\models (D, z = M) \quad \Gamma) \quad \text{p} \quad \text{s}\,(\models (D, z = M) \quad (z \quad \phi))$$

n ons qu n u str t on s t or λ u us t r s, t s op r t on n t on r s t t n t on t on .. □

n n pro s st or Lam s or Λ_P. s us s t s propo s t ons, n v u nts t or $\Gamma \vdash M \quad \phi$ n $\Gamma \vdash D \quad \Delta$. n r n t nt pro s st or Lam n t t Λ_P s t pro s st or r urs v r t ons. ot t t

- pro ru s ( ) n (?) or t n unt r t ons r t s . nt t r s no r n t n t or n unt no , on u t s s n $\phi = \psi \rightarrow \chi$

Ξ

n n

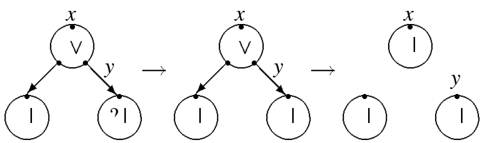
( )

$$\partial[\![D,E]\!] = (X \cup X', Y \cup Y', Z \cup Z', f \cup f')$$
$$\partial[\![\nu x\,.\,D]\!] = (X \setminus \{x\}, Y \cup \{x\}, Z, f)$$

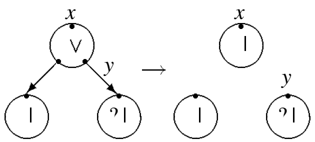
r $\partial[\![D]\!] = (X,Y,Z,f)$, $\partial[\![E]\!] = (X',Y',Z',f')$ n X, Y, X' n Y' r s
o nt− □

n nsotttss nt s s u str t or $\equiv$−

For p , o t r u t on



ut not



ot t t n t r o stor p t o

$$(x = y \vee z, y = \lambda w.M) \mapsto (x = \mathsf{I}, y = \lambda w.M)$$

s n v to ons r t s s n $x = z$, $y = z$, n $x \neq z \neq y$

DEF $\mapsto_c$ s v n o s

(BUILD)	$x = (\mathsf{rec}\, D \,\mathsf{in}\, M) \mapsto_c \mathsf{rec}\, D \,\mathsf{in}\, (x = M)$
(∇TRAV)	$x = \nabla y, y = {}^{?}M \mapsto_c x = \nabla y, y = M$
(TRAV)	$x = y \quad z, y = {}^{?}M \mapsto_c x = y \quad z, y = M$
($\vee$TRAV)	$x = y \vee z, y = {}^{?}M \mapsto_c x = y \vee z, y = M$
(∇UPD)	$x = \nabla y, y = \lambda w.M \mapsto_c x = \lambda w.M, y = \lambda w.M$
(UPD)	$x = y \quad z, y = \lambda w.M \mapsto_c x = M[z/w], y = \lambda w.M$
($\vee$UPDa)	$x = y \vee z, y = \lambda w.M, z = N \mapsto_c x = \mathsf{I}, y = \lambda w.M, z = N$
($\vee$UPDb)	$x = y \vee y, y = \lambda w.M \mapsto_c x = \mathsf{I}, y = \lambda w.M$
($\vee$UPDc)	$x = y \vee x, y = \lambda w.M \mapsto_c x = \mathsf{I}, y = \lambda w.M$

n stru tur ru s

(L) $\dfrac{D \mapsto_c E}{D, F \mapsto_c E, F}$ (R) D

– *For closed* D

– $\leq$ *is a partial order*
– $D \leq \equiv E$ *iff* $D \equiv \leq E$
– *If* $D \rightarrow_{\gamma} \rightarrow_{c} E$ *then* $D \rightarrow_{c} \rightarrow_{\gamma} E$
– *If* $D \leq \rightarrow_{c} E$ *then* $D \rightarrow_{c}^{\leq} \leq E$
– *If* $D \leq \rightarrow_{\gamma} E$ *then* $D \rightarrow_{\gamma} \leq E$
– *If* $D \rightarrow E$ *then* $D \rightarrow_{c}^{*} \rightarrow_{\gamma}^{*} \leq E$
– *If* $D \rightarrow^{*} E$ *then* $D \rightarrow_{c}^{*} \rightarrow_{\gamma}^{*} \leq E$

F–

–B n t on $\leq$ s r v –B n u t on on t proo $D \leq E$ n
s o t t $D \leq E \leq D$ t n $D =$

$$
\begin{aligned}
&\longrightarrow_c \nu\vec{x}\vec{y}\,.\,(F', z = M, G) && \vee\text{TRAV}\\
&\longrightarrow_c \nu\vec{x}\vec{y}\,.\,(F', z = M, H) && \vee\text{UPD}a\\
&\equiv \nu\vec{x}\,.\,(\nu\vec{y}\,.\,(F', z = M), H) && \nu\text{MIG}\\
&\leq \nu\vec{x}\,.\,(\nu\vec{y}\,.\,(F', z = M), H) && \text{D n } \leq\\
&\equiv \nu\vec{x}\,.\,(F, H) && \text{Eqn}\\
&\equiv E && \text{Eqn}
\end{aligned}
$$

us, $D \longrightarrow_c^* \longrightarrow_\gamma^* \leq E$ –

(OTHERS) ot r o s r o s $\mapsto_c$, n so $D \longrightarrow_c^* \longrightarrow_\gamma^* \leq E$ –

– t $D \longrightarrow^n E$, n pro n u t on on n

(

– *If* $D \equiv (D', x = \lambda w.M) \rightarrow_c E$ *then* $E' \equiv (E', x = \lambda w.M)$

– *If* $D \equiv (D', x = ?M) \rightarrow_c E$ *then* $E \equiv (D', x = M)$

or $E \equiv (E', x = $ [illegible]

n ropos t on – t r

- v

$$H \equiv (G, \mathsf{local}\, K \;\mathsf{in}\; x = M)$$

n s

$$
\begin{aligned}
E & \\
&\equiv \nu\vec{x}\,.\,(G, J) && \text{Eqn} \\
&\equiv \nu\vec{x}\,.\,(G, \mathsf{local}\, K \;\mathsf{in}\; x = M) && \text{Eqn} \\
&\equiv \nu\vec{x}\,.\,H && \text{Eqn} \\
&\equiv F && \text{Eqn}
\end{aligned}
$$

- or v

$$H \equiv (L, x = \mathsf{rec}\, K \;\mathsf{in}\; M$$

n or n N

$$(G, x$$

r u t on s n n or r to $_{\mathrm{v}}$ u t x

– *If* $D \vdash x \prec y$ *then* $D, E \vdash x \prec y$.
– *If* $\nu x . D \vdash y \prec z$ *then* $D \vdash y \prec z$.
– *If* $x \neq y \neq z$ *w is fresh and* $D \vdash x \prec z$ *then* $[w/y]D[w/y] \vdash x \prec z$.

F– n u t ons on t pro $\prec$– □

n n s o t t $D \rightarrow_x E$ t r s r u t on on t x sp n D

– $D \rightarrow_x E$ *iff* $D \equiv \nu\vec{x} . F$ $E \equiv \nu\vec{x} . G$ $F \rightarrow_y G$ *is an axiom and* $F \vdash x \prec y$.

F–

⇒ An n u t on on t pro $D \rightarrow_x E$–
⇐ An n u t on on t pro $F \vdash x \prec y$– □

– *If* $D \vdash x \prec y$ *and* $D \rightarrow_y E$ *then* $D \rightarrow_x E$.

F–B ropos t on $D \equiv \nu\vec{x} . F$, $E \equiv \nu\vec{x} . G$, $F \vdash y \prec z$ n $F \rightarrow_z G$ s n
n – – – – n

n so

$$
\begin{aligned}
&D \\
&\quad \equiv \nu\vec{x}\,.\,(F, G) && \text{Eqn} \\
&\quad \equiv \nu\vec{x}\,.\,(F
\end{aligned}
$$

$\equiv \nu y\vec{w}.(G',H)$ (νMIG) n (νSWAP)

n

$$\nu\vec{w}.(G',H)$$
$$\rightarrow_c \nu\vec{w}.(G',I) \quad \text{Eqn}$$
$$\equiv E' \quad \text{Eqn}$$

- or v

$$I \equiv \nu y.I' \qquad E' \equiv \nu\vec{w}.(G,I')$$

so n s s o t t ou v $H \mapsto_c I$, n t tt on poss t s (BUILD) n s

$$D \equiv \nu\vec{w}.(G, z = \mathsf{rec}\, F \;\mathsf{in}\; M)$$
$$E \equiv \nu\vec{w}.(G, \mathsf{local}\, F \;\mathsf{in}\; z = M)$$
$$E' \equiv \nu\vec{w}.(G,I')$$
$$\nu y.I' \equiv \mathsf{local}\, F \;\mathsf{in}\; z = M$$

−B ropos t on −

$$D \equiv \nu\vec{y}.(G,H) \qquad E \equiv \nu\vec{y}.(G,I) \qquad H \mapsto_c I \text{ s n o}$$

n ropos t ons n − v

$$D' \equiv \nu\vec{y}.D''$$
$$(D'', x = M) \equiv (G,H)$$
$$E' \equiv \nu\vec{y}.E''$$
$$(E'', x = M) \equiv (G,I)$$

n ropos t ons − n − t r

- v

$$G \equiv (G', x = M) \qquad D'' \equiv (G',H) \qquad E'' \equiv (G',I)$$

so or n N

$$D', x = N$$
$$\equiv (\nu\vec{y}.D''), x = N \quad \text{Eqn}$$
$$\equiv (\nu\vec{y}.(G',H)), x = N \quad \text{Eqn}$$
$$\mapsto_c (\nu\vec{y}.(G',I)), x = N \quad \text{Eqn}$$
$$\equiv (\nu\vec{y}.E''), x = N \quad \text{Eqn}$$
$$\equiv E', x = N \quad \text{Eqn}$$

- or v

$$H \equiv (H', x = M) \quad D'' \equiv (G,H') \quad I \equiv (I', x = M) \quad E'' \equiv (G,I')$$

n s n s s o ou v $H \rightarrow_c I$, n t t t r

- $G,H \rightarrow_x G,I$ n so $D \rightarrow_x E$ −
- For n N, $H', x = N \rightarrow I', x = N$, n so $D', x = N \rightarrow E', x = N$ −

−B ropos t on

$$D \equiv \nu\vec{x}.F \qquad E \equiv \nu\vec{x}.G \qquad F \vdash y \prec z \qquad F \rightarrow_z G \text{ s n o}$$

n n α on rt so t t $x \not\in x$ n ropos t on t r

- v

$$\vec{x} = \vec{y}w\vec{z} \qquad D' \equiv \nu\vec{y}\vec{z}.[x/w]F[x/w]$$

n so

E

$$\equiv \nu\vec{x}.G \quad \text{Eqn}$$
$$\equiv \nu\vec{y}w\vec{z}.G \quad \text{Eqn}$$
$$\equiv \nu\vec{y}\vec{z}.[x/w]G[x/w] \quad (\alpha) \text{ n } (\nu\text{SWAP})$$

$$[x/w]F[x/w] \rightarrow_z [x/w]G[x/w]$$

ropos t on

$$[x/w]F[$$

- $D \equiv D$

so (∇IND), $D \to_x E$, n so $D \to_x \to_c F$−

- $D \to_x E$, n so $D \to_x \to_c F$−

(IND) s s r−

(∨IND) s s r− □

− *For closed D if x is tagged in D and* $D \to_c^* E$ *then* $D \to_x^* \to_{\neg x}^* E$.

F− t $D \to_c^n E$, n pro n u t on on n−

- $n =$ t n $D \equiv E$ so $D \to_x^* \to_{\neg x}^* E$−
- $n >$ t n $D \to_c F \to_c^{n-} E$, n ropos t on − x s t n F so n u t on $F \to_x^* \to_{\neg x}^* E$, so ropos t on $D \to_x^* \to_c \to_{\neg x}^* E$, n so $D \to_x^* \to_{\neg x}^* E$− □

− *For closed D if x is tagged in D*

$$
\begin{aligned}
&{}_{\gamma}\ \nu\vec{x}\,.\,(I, \nu(\mathsf{wv}\,G)\,.\,G) && \gamma\\
&\equiv\ \nu\vec{x}\,.\,H && \text{Eqn}\\
&\equiv\ E && \text{Eqn}
\end{aligned}
$$

us $D \rightarrow_x \quad _\gamma E$ −

□

− *For closed D if $D \rightarrow E$ then $D{\Downarrow_x}$ iff $E{\Downarrow_x}$*

F −

$\Rightarrow$ $D \rightarrow_c E$, t n ropos t on , $E{\Downarrow_x}$ −

$D \rightarrow_\gamma E$, t n ropos t ons n −

$$
\begin{array}{l}
\mathsf{tag}_x D \rightarrow_x \cdots \rightarrow_x F\\
\downarrow^{\leq}_{\gamma}\\
\mathsf{tag}_x E
\end{array}
$$

- t r s $F_i = (x_i = M_i)$, n $w_i = \varepsilon$ −

  n

- For i su t t $D[\vec{x}/\vec{z}] \vdash x \sim x_i$, $(x_i = M_i[\vec{x}/\vec{z}]) \rightarrow_c \nu\vec{w}_i . F_i[\vec{x}/\vec{z}]$ n so $E \rightarrow_c^* F[\vec{x}/\vec{z}]$ −
- r , $D[\vec{y}/\vec{z}] \rightarrow_c^* F[\vec{y}/\vec{z}]$ −
- t $\mathcal{R}$ ν ss $D[\vec{x}/\vec{z}]$ s u t on su t t $\vec{x}\ \mathcal{R}\ \vec{y}$ − n t $\mathcal{R}'$ t s st r t on ont n n $\mathcal{R}$ su t t $\vec{w}\vec{w}\vec{w}_i\vec{w}_i\ \mathcal{R}\ \vec{w}\vec{w}_j\vec{w}_j\vec{w}$ − n s o $\mathcal{R}'$ s ν ss $(G, x = M, F, \ldots, F_n)[\vec{x}/\vec{z}]$ s u t on, n so $F[\vec{x}/\vec{z}] \vdash \vec{x} \sim \vec{y}$ − □

n (νMIG)

$$\nu\vec{x}\,.\,(D, \mathsf{local}\, G \,\mathsf{in}\, x = M', \mathsf{local}\, H \,\mathsf{in}\, y = N')$$
$$\equiv \nu\vec{x}\,.\,\nu(\mathsf{wv}\, G)\,.\,\nu(\mathsf{wv}\, H)\,.\,(D, G, H, x = M', y = N')$$

n ro t n t on s u t on

$$\nu\vec{x}\,.\,\nu(\mathsf{wv}\, G)\,.\,\nu(\mathsf{wv}\, H)\,.\,(D, G, H, x = M', y = N') \vdash x \sim y$$

so n u t on

$$\nu\vec{x}\,.\,\nu(\mathsf{wv}\, G)\,.\,\nu(\mathsf{wv}\, H)\,.\,(D, G, H, x = \nabla y, y = N')\Downarrow_z$$

n so

$$\begin{aligned} &\nu\vec{x}\,.\,(D, x = \nabla y, y = N) \\ &\equiv \nu\vec{x}\,.\,(D, x = \nabla y, y = \mathsf{rec}\, H \,\mathsf{in}\, N') && \text{Eqn} \\ &\rightarrow \nu\vec{x}\,.\,(D, x = \nabla y, \mathsf{local}\, H \,\mathsf{in}\, y = N') && \text{B D} \\ &\quad \nu\vec{x}\,.\,(D, \mathsf{local}\, G \,\mathsf{in}\, \varepsilon, x = \nabla y, \mathsf{local}\, H \,\mathsf{in}\, y = N') && \gamma \\ &\equiv \nu\vec{x}\,.\,\nu(\mathsf{wv}\, G)\,.\,\nu(\mathsf{wv}\, H)\,.\,(D, G, H, x = \nabla y, y = N') && \nu\text{MIG} \end{aligned}$$

n so Equ t on n ropos t on

$$\nu\vec{x}\,.\,(D, x = \nabla y, y = N)\Downarrow_z$$

ot r s s r s r

$\Leftarrow$ sy wvi0 238.2.240113 12(m)-8302154(i)-5.01912(l)-5f 0 3 23364 11 -1 0 3.1126!

$$\bot \circ f = \bot$$

n so un or t

$$\mathsf{fix}(\mathsf{set}\,X g) \circ f = \mathsf{fix}(\mathsf{set}\,X g)$$

Fro t s t s s to s o n u t on on D t t $[\![D]\!] = [\![D]\!] \circ f$ −

− $(\mathsf{wv}[\![D]\!] \subseteq \mathsf{wv}\,D)$ An n u t on on D −

$(\mathsf{wv}[\![D]\!] \supseteq \mathsf{wv}\,D)$ $\mathsf{wv}[\![D]\!]\,\mathsf{wv}\,D$ t n n $x \in \mathsf{wv}\,D$ n $x \notin \mathsf{wv}[\![D]\!]$ − n

$$\begin{aligned} &\top \\ &= \mathsf{read}\,x \circ (x \;\; = \top) && \text{ropn} \\ &= \mathsf{read}\,x \circ [\![D]\!] \circ (x \;\; = \top) && x \notin \mathsf{wv}[\![D]\!] \\ &= \mathsf{read}\,x \circ [\![\end{aligned}$$

$= \mathsf{read}\, x \circ f$ $\quad f = g \circ f$

$x \notin X$ t n

$\mathsf{read}\, x \circ (\mathsf{set}\, X g)^{n+} \perp \circ f$
$= \mathsf{read}\, x \circ (\mathsf{set}\, X g)((\mathsf{set}\, X g)^{n} \perp) \circ f$ $\quad$ D n f^n
$= \mathsf{read}\, x \circ f$ $\quad$ ropn

us $(\mathsf{set}\, X g)^{n+} \perp \circ f \leq f$

us

$$
\begin{aligned}
& f = g \circ f \\
& \Rightarrow \bigvee\{(\mathsf{set}\, X g)^{n} \perp \circ f \mid n \;\mathsf{in}\; \omega\} \leq f && \text{A o} \\
& \Rightarrow \bigvee\{(\mathsf{set}\, X g)^{n} \perp \mid n \;\mathsf{in}\; \omega\} \circ f \leq f && \circ \text{ s ont nuous} \\
& \Rightarrow \mathsf{fix}(\mathsf{set}\, X g) \circ f \leq f && \text{D n fix}
\end{aligned}
$$

For p wv $f = X$, wv $g = Y$ n $X \cap Y = \emptyset$ t n v p rt

$$\mathsf{fix}(\mathsf{set}(X \cup Y)(f \circ g)) = f \circ \mathsf{fix}(\mathsf{set}(X \cup Y)(f \circ g))$$

n so t o

$$\mathsf{fix}(\mathsf{set}\, X f) \circ \mathsf{fix}(\mathsf{set}(X \cup Y)(f \circ g)) \leq \mathsf{fix}(\mathsf{set}(X \cup Y)(f \circ g))$$

r

$$\mathsf{fix}(\mathsf{set}\, Y g) \circ \mathsf{fix}(\mathsf{set}(X \cup Y)(f \circ g)) \leq \mathsf{fix}(\mathsf{set}(X \cup Y)(f \circ g))$$

us

$$
\begin{aligned}
& \mathsf{set}(X \cup Y)(\mathsf{fix}(\mathsf{set}\, X f) \circ \mathsf{fix}(\mathsf{set}\, Y g))(\mathsf{fix}(\mathsf{set}(X \cup Y)(f \circ g))) \\
& = \mathsf{fix}(\mathsf{set}\, X f) \circ \mathsf{fix}(\mathsf{set}\, Y g) \circ \mathsf{fix}(\mathsf{set}(X \cup Y)(f \circ g)) && \text{ropn} \\
& \leq \mathsf{fix}(\mathsf{set}\, X f) \circ \mathsf{fix}(\mathsf{set}(X \cup Y)(f \circ g)) && \text{Eqn} \\
& \leq \mathsf{fix}(\mathsf{set}(X \cup Y)(f \circ g)) && \text{Eqn}
\end{aligned}
$$

☛ $x \in$ wv D t n

$[\![($rec D in M

$$= [\![D]\!]$$

− Assu [redacted]

[redacted] $= M)$ $(w$ $\psi \rightarrow \chi)$

[redacted] $)\Downarrow_w$ so ropos t on −

[redacted] $(D, w = M, x = M)\Downarrow_x$

[redacted] n $(z = x \quad y) \sqsubseteq E \sqsupseteq (D, w = M, x = M)$ t r

[redacted] $= w$ or $z = x$, so $M = x \quad y$, so $(D, w = M, x = M)\Uparrow_x$, s on

[redacted] s t on , n n F su t t

$(F, w$ [redacted] $) = E$ $(F$

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F–

–An n u t on on ϕ– on u t s s n $\phi = \psi \rightarrow \chi$–

$\Rightarrow$ $\models D$ $(x$ $\psi \rightarrow \chi)$ t n $D\Downarrow_x$ so ropos t on $\nu w.\, D\Downarrow_x$–For n $(z = x \quad y) \sqsubseteq E \sqsupseteq (\nu w.\, D)$, t v r s , ropos t on , n n $F \sqsupseteq (z = x \quad y)$ su t t

$$E \equiv \nu v.\, F \qquad F \sqsupseteq [v/w]D[v/w]$$

so ropos t on

$$\models [v/w]D[v/w] \quad (x \quad \psi \rightarrow \chi)$$

n so

$$\models E \quad (y \quad \psi)$$
$$\Rightarrow \models \nu v.\, F \quad (y \quad \psi)$$

• $w = x$ t n n n rs $\vec{y}$ n I su t t

$$H \equiv \nu\vec{x}\vec{y}\,.\,(F, G, I, w = M, z = w \; y)$$

so t $\vec{w} = wvG$, n t v n $\vec{v}$ rs n s n $\models D\ (x\ \psi \to \chi)$, ropos t on

$$\nu\vec{x}\,.\,(F, v = \mathsf{rec}\,G\,\mathsf{in}\,M)[v/w]\quad(v\ \psi \to \chi)$$

n ro t n t on $\sqsubseteq$

$$(z = v\ y)$$
$$\sqsubseteq \nu\vec{x}\,.\,(F[v/w], G, I, v = (\mathsf{rec}\,G\,\mathsf{in}\,M)[v/w],$$
$$w = M[v/w], z = v\ y)$$
$$\sqsupseteq \nu\vec{x}\,.\,(F, v = \mathsf{rec}\,G\,\mathsf{in}\,M)[v/w]$$

n

$\models H\ (y\ \psi)$
$\Rightarrow \models \nu\vec{x}\vec{y}\,.\,(F, G, I, w = M, z = w\ y)\ (y\ \psi)$ Eqn
$\Rightarrow \models (F, G, I, w = M, z = w\ y)\ (y\ \psi)$ ropn
$\Rightarrow \models (F, G, I, [v\vec{v}/w\vec{w}]G[v\vec{v}/w\vec{w}],$
$v = M, w = M, z = w\ y)\ (y\ \psi)$ ropn
$\Rightarrow \models (F[v/w], G, I, [v\vec{v}/w\vec{w}]G[v\vec{v}/w\vec{w}],$
$v = M[v/w], w = M[v/w], z = v\ y)\ (y\ \psi)$ ropn
$\Rightarrow \models (F[v/w], G, I, v = (\mathsf{rec}\,G\,\mathsf{in}\,M)[v/w],$
$w = M[v/w], z = v\ y)\ (y\ \psi)$ n n
$\Rightarrow \models (F[v/w], G, I, v = (\mathsf{rec}\,G\,\mathsf{in}\,M)[v/w],$
$w = M[v/w], z = v\ y)\ (z\ \chi)$ Eqns n
$\Rightarrow \models H\ (z\ \chi)$ r

us $\models E\ (x\ \psi \to \chi)$.

• $x \neq w \neq z$ t n t pro s s r.

(OTHER) $D \to_c E$ s pro t out B D t n n s o t t

$$D \sqsubseteq D' \quad \text{p} \quad \text{s}\ D' \to_c E' \sqsupseteq E$$
$$E \sqsubseteq E' \quad \text{p} \quad \text{s}\ D \sqsubseteq D' \to_c E'$$

n $\models D\ (x\ \psi \to \chi)$ t n $D\Downarrow_x$ so ropos t on , $E\Downarrow_x$. n or n $(z = x\ y) \sqsubseteq F \sqsupseteq E$, n n G su t t

$$F \equiv (G, z = x\ y)$$

n t w r s , so

$$(w = x\ y) \sqsubseteq (G, w = x\ y, z = x\ y) \sqsupseteq E$$

n n n $H \sqsupseteq D$ su t t

$$H \to_c F$$

n

$\models F\ (y\ \psi)$
$\Rightarrow \models (G, z = x\ y)\ (y\ \psi)$ Eqn
$\Rightarrow \models (G, z = x\ y, w = x\ y)\ (y\ \psi)$ ropn
$\Rightarrow \models (H, w = x\ y)\ (y\ \psi)$ n n
$\Rightarrow \models (H, w = x\ y)\ (w\ \chi)$ $\models D\ (x\ \psi \to \chi)$
$\Rightarrow \models (G, z = x\ y, w = x\ y)\ (w\ \chi)$ n n
$\Rightarrow \models (G, z = x\ y, w = x\ y)\ (z\ \chi)$ ropn
$\Rightarrow \models (G, z = x\ y)\ (z\ \chi)$ ropn
$\Rightarrow \models F\ (z\ \chi)$ Eqn

us or n $(z = x\ y) \sqsubseteq F \sqsupseteq E$

$$\models F\ (y\ \psi) \Rightarrow \models F\ (z\ \chi)$$

so $\models E\ (x\ \psi \to \chi)$.
ot r r t on s s o n s r. □

3.11 Full abstraction

n t s s t on, s o t t o **D** s u str t or on urr nt r p r u t on. s nst t on urr nt r p r u t on s t s u str t o s t ost out r ostr u t on, n so on urr nt r p r u t on s t t s o put t on po r s t ost out r ostr u t on. s pro o o s t s stru tur s t on .

• s o t t $\Gamma \vdash D\ \Delta$ $[\![\Delta]\!] \leq [\![D]\!][\![\Gamma]\!]$, t us s o t t pro s st s soun n o p t or t not t on s nt s. s s ropos t on , t r p r u t on qu nt ropos t on .

• t n s o t t $\Gamma \vdash D\ \Delta$ t n $\Gamma \models D\ \Delta$, n t t $\Gamma \models D\ \Delta$ t n $[\![\Delta]\!] \leq [\![D]\!][\![\Gamma]\!]$. us t t r pr s nt t ons t o r qu nt. s s ropos t on , t r p r u t on qu nt ropos t on .

• F n , s o t t u st t on s n pro n t t r o pr s nt t ons to qu nt. s s ropos t on , t r p r u t on qu nt ropos t on .

us, ABA Y n G s t n qu s n pt to r p r u t on.

.

. $\Gamma \vdash M\ \phi$ *iff* $[\![\phi]\!] \leq [\![M]\!][\![\Gamma]\!]$.
. $\Gamma \vdash D\ \Delta$ *iff* $[\![\Delta]\!] \leq [\![D]\!][\![\Gamma]\!]$.

F—

D E $\Rightarrow$ v to prov t ru s $\Gamma \vdash M$ ϕ n $\Gamma \vdash D$ Δ soun —For p , to prov ( ), $[\![\Delta]\!] \leq [\![x = M]\!][\![\Gamma]\!]$ n $[\![\phi]\!] \leq [\![M]\!][\![\Delta]\!]$ t n

$$
\begin{aligned}
& [\![x \quad \phi]\!] \\
& \quad \leq (x = [\![M]\!])[\![\Delta]\!] && \text{H pot s s} \\
& \quad \leq (x = [\![M]\!])([\![x = M]\!][\![\Gamma]\!]) && \text{H pot s s} \\
& \quad = [\![x = M]\!][\![\Gamma]\!] && \text{ropn —}
\end{aligned}
$$

ot r s s r s r—

C E E E $\Leftarrow$ An n u t on on M n D—For p , $x \neq y$ n

$$[\![\phi]\!] \leq [\![x \quad y]\!][\![\Gamma]\!]$$

t n t r $[\![\phi]\!] = \perp$, so $\vdash \phi = \omega$ n so $\Gamma \vdash x \quad y \quad \phi$, or

$$
\begin{aligned}
& [\![\phi]\!] \leq [\![x \quad y]\!][\![\Gamma]\!] \\
& \quad \Rightarrow [\![\phi]\!] \leq \mathsf{apply}[\![\Gamma(x)]\!][\![\Gamma(y)]\!] && \text{D n } [\![x \quad y]\!] \\
& \quad \Rightarrow [\![\Gamma(y) \rightarrow \phi]\!] \leq [\![\Gamma(x)]\!] && \text{ropn —} \\
& \quad \Rightarrow \vdash \Gamma(x) \leq \Gamma(y) \rightarrow \phi && \text{ropn} \\
& \quad \Rightarrow \vdash \Gamma \leq x \quad \Gamma(y) \rightarrow \phi, y \quad \Gamma(y) && \text{D n } \leq \\
& \quad \Rightarrow \Gamma \vdash x \quad y \quad \phi && (\leq)
\end{aligned}
$$

$$(z = x \quad y) \sqsubseteq E \sqsupseteq (D, x = \lambda w.\, M)$$

• $\models D$ $(x$ $\phi \to \psi)$ t n $D\Downarrow_x$ so Coro r [[D]]$\sigma x \neq \bot^{-}$A so, or
r s y n z

true

$\Rightarrow$

4 Conclusions

nt sp p r, v n v st t t r t ons p t nt s nt not on *full abstraction* n t p nt t on t n qu *concurrent graph reduction*—
v s o n t t

- Con urr nt r p r u t on n v n s p op r t on pr s nt t on n t st BE Y n B D s *chemical abstract machine*, n E s *polyadic π calculus*—
- t n qu s AB A Y n G s *lazy λ calculus* n

urs v r t ons, o v r s or s– AD H so n v st t s t
r t ons p t n r p r u t on n t $\mathbf{D}_\infty$ o t unt p λ u us
s BA E D EG , or or t s, top s t r p up
E E n AB A Y n G –

BA A D EG *et al.* r s r o or on *term graph rewriting*,
ntro u BA A D EG *et al.* , n surv E A AY *et al.*
n t ot r p p rs n EE *et al.* s EE *et al.*, oo – r
r p s r v r s r to r t ons, ut r root , n , oss B – A –

H HA A A D EA A – Anot r ppro to t op r t on s nt s r p r u t on s H HA A n EA A s AZY CF HA t n s s CF t let r t ons – s s v n st p op r t on s nt s t or n our s nt

$$(\mathsf{let}\, D\, \mathsf{in}\, M) \Downarrow (\mathsf{let}\, E\, \mathsf{in}\, N)$$

s s nt s s s r to ours n A CHB Y s pt t t

- AZY CF HA s t p n u n s onstru tors n onstru tors or oo ns n n tur nu rs –
- n let pr ss ons r n us r t r t n rec pr ss ons t s n t s or po nts os so s r n n or t on

$$(\mathsf{let}\, D\, \mathsf{in}\, \mathsf{let}\, x = (\mu x.\, M)\, \mathsf{in}\, M) \Downarrow (\mathsf{let}\, E\, \mathsf{in}\, N)$$

F n n proo t n qu t t s po u nou to s o u str t on or on urr nt r p r u t on ut o s not r on on s n s s s to qu t u t

Y ED λ CA C proo s n n t s p p r r on or t unt p λ u us t r urs v r t ons non str t un t on n u s r us n pr t r t p n v t p onstru tors n onstru tors usu n t or o p tt rn t n

u onstru tors n onstru tors ou to t λ u us t r urs v r t ons For p t pro u t t p $T \times U$ t onstru tors n onstru tors

$$\mathsf{pair} \quad T \to U \to (T \times U) \qquad \mathsf{fst} \quad (T \times U) \to T \qquad \mathsf{snd} \quad (T \times U) \to U$$

ou to t λ u us t r urs v r t ons s

$$M \quad = \cdots \mid \mathsf{pair}\, x y \mid \mathsf{fst}\, x \mid \mathsf{snd}\, x$$

t t op r t on s nt s or

⊥, l or r choose unction ou to t λ u us t r urs v
r t ons s

$$M ::= \cdots \mid \mathsf{choose}\, x y$$
$$D ::= \cdots \mid \mathsf{o} = \bot \mid \mathsf{o} = \mathsf{l} \mid \mathsf{o} = \mathsf{r}$$

t t op r t on s nt s v n

$$x = \mathsf{choose}\, yz, y = {?}M \mapsto x = \mathsf{choose}\, yz, y = M$$
$$x = \mathsf{choose}\, yz, z = {?}M \mapsto x = \mathsf{choose}\, yz, z = M$$
$$\mathsf{o} = \bot, x = \mathsf{choose}\, yz, y = \lambda w.M \mapsto \mathsf{o} = \mathsf{l}, x = \mathsf{choose}\, yz, y = \lambda w.M$$
$$\mathsf{o} = \bot, x = \mathsf{choose}\, yz, z = \lambda w.M$$

E D — *Combinator Graph Reduction A Congruence and its Applications* D t s s or n v rs t

AC A E — *Categories for the orking Mathematician* Gr u t ts n t t s pr n r r

— Fu str t s nt s t p λ u *Theoret Comput Sci*

— *Communication and Concurrency* r nt H

— po π u us tutor n *Proc International Summer School on Logic and Algebra of Speci cation* r to r o

H u us o s pro r n n s D ss rt t on

YC F A — *Abstract Interpretation and Optimising Transformations for Applicative Programs* D t s s E n ur n v rs t D pt Co put r n

G C H — *The Lazy Lambda Calculus An Investigation into the Foundations of Functional Programming* D t s s p r Co on on n v rs t

G C H — on t r n s n un t on s tt n n *Proc LICS* p s EEE Co put r o r ss

EY E — *The Implementation of Functional Programming Languages* r nt H

CE B C — *Basic Category Theory for Computer Scientists* pr ss

G — CF ons r s pro r n n *Theoret Comput Sci*

G — Do ns v non ous tp

H HA A n EA A — An qu t op r t on s nt s s r n u t on n *Proc ESOP*

E H — E p t su st tut on nt s not D D n v rs t Cop n n

C D — Do ns or not t on s nt s n E E n CH D E tors *Proc ICALP* p s pr n r r C

EE A E n A EE E E C D tors *Term Graph Rewriting Theory and Practice* n n ons

Y E — *Denotational Semantics The Scott Strachey Approach to Programming Language Theory* r ss

D — A n p nt t on t n qu or pp t v n u s *Software Practice and Experience*

D — r n A non str t un t on n t po orp t p s n *Proc IFIP Conf Functional Programming Languages and Computer Architecture* pr n r r C

AD H C — *Semantics and Pragmatics of the Lambda Calculus* D t s s or n v rs t

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E ,
s u t on,
s u t on, ν ss,
s t or ,
split,